Performance Analysis of 802.22 based Cognitive Radio Networks

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Abstract— **We present an analytical model that predicts the throughput of a cognitive radio network based on renewal theory. The analytical model also assesses the impact of two models of the incumbent operation (idle / busy), per constant and exponential distributions, on the throughput of the cognitive radio network. The theoretical model for exponentially distributed idle and busy times is developed by taking the primary user's operation as a continuous-time markov process. Finally, we show that simulation results compare well against the analytical model's prediction of the throughput of the cognitive radio network.**

***Keywords*— Cognitive radio, IEEE 802.22, renewal reward, throughput, simulation, time-lag, NetSim**

I. Introduction

IEEE formed a working group (IEEE 802.22) to develop an air interface for opportunistic secondary access to the TV spectrum. The standard developed by 802.22 group, better known as Cognitive Radio. Cognitive networks (CR), enables efficient utilization of the scarce spectrum by allowing spectrum sharing between a licensed primary network and a secondary network. Spectral utilization can be further enhanced by allowing users from crowded bands to bleed off into nearby empty bands. A CR can intelligently detect whether any portion of the spectrum is not in use, and can temporarily use it without interfering with the transmissions of other users. A CR can sense its environment and, without the intervention of the user, can adapt to the user's communications needs. This optimizes the use of available radio-frequency (RF) spectrum while minimizing interference to other users and thus reduces spectrum scarcity

This paper is organized as follows. In Section II, we provide the details about the 802.22 protocol. In Section III, we state the key assumptions on which the analysis is based. We describe the system model in Section IV. The analytical model is provided in Section V. Numerical results follow in Section VI. Finally, this paper is concluded in Section VII.

II. The 802.22 Protocol

A. Devices in Cognitve Radio Network

The cognitive radio network can have the following types of devices:

- Base station (BS) is generalized equipment set providing connectivity, management and control of the customer premise equipment (CPE).
- Primary Users (Incumbents) are licensed transmission systems operating in the television bands.
- Secondary Users (CPEs) exploit cognitive radio techniques, to ensure non-interfering co-existence with the primary users.

B. Superframe And Frame Structure

A frame is a basic time division in the 802.22 protocol. A superframe is a group of 16 consecutive frames. Each frame is 10 ms long.

The cognitive radio network has two types of packet transfer: downstream – from the Base Station to the CPE and upstream, which is vice-versa. The following packets are transmitted in sequence in each frame from the BS to CPE:

1. Preambles: Superframe preamble (only in first frame of a superframe) and Frame preamble
2. Superframe Control Header (SCH): It is transmitted only in first frame of every superframe. It contains information including the Quiet Period scheduling. In order to associate with a base station, a CPE must receive the SCH to establish communication with the BS.
3. Frame Control Header (FCH): The FCH specifies the burst profile and the length of either the DS-MAP, if transmitted, or the US-MAP.
4. Downstream Map (DS-MAP): DS-MAP message defines the access to the downstream information.
5. Upstream Map (US-MAP): It defines the slot allocation to each CPE for upstream transmission.
6. Channel Descriptors: upstream (UCD) and downstream (DCD).
7. The control packets are followed by downstream bursts (DS-BURSTS) which contain the data packets.

This marks the end of the downstream subframe. Then we have the Transmit/Receive Transition Gap (TTG) to allow the CPE to switch between the receive mode and transmit mode. No data transfer occurs in this period.

TTG is followed by upstream subframe. Initially some time is reserved for Ranging, Bandwidth (BW) Request and Urgent Coexistence Situation (UCS) slots. Next are the upstream (US) bursts, which carry data packets from a CPE to the BS. They might be followed by the Intra Frame Quiet period (if specified in the SCH), during which the base station has scheduled a cessation of all transmission in its cell for the purpose of sensing. They are succeeded by Receive/Transmit Transition Gap (RTG) to allow the BS to switch between its receiving mode and transmit mode. Data transfer is prohibited during the RTG.

III. MODELLING ASSUMPTIONS FOR THE 802.22 PROTOCOL

In order to obtain analytically tractable models, we have made some simplifications, which we will now discuss.

For simplicity, we consider only downlink traffic. This condition allows the exclusion of the effect of cochannel interference, as well as coexistence mechanisms. We also assume that packets can't be split over symbols. We have reserved one downstream symbol for the control packets, namely FCH, USMAP, DSMAP, UCD and DCD.

A single-cell configuration with a single secondary user (SU) and a primary user (PU) is used. There is only one operating channel at a time for all SUs.

An implicit assumption is time-synchrony between PU and SU, because it is a well-established classical assumption in the theoretical analysis. Furthermore, we assume that the subchannel throughput is, on the average, constant.

We also neglect the errors due to fading and noise. This effect is modeled via "*keep out distance*". If the PU is beyond the "*keep out distance*", we take it that no interference can take place with the SU. But the SU might make wrong decisions about the presence of PU based on Probability of False Alarm and Probability of detection.

Although we have non-zero quiet period duration, the channel sensing in our model is instantaneous and is done only in the beginning of each quiet period.

In case no operating channel is available to the secondary users, the channels will be sensed after every one second.

IV. SYSTEM MODEL

A. System Configuration

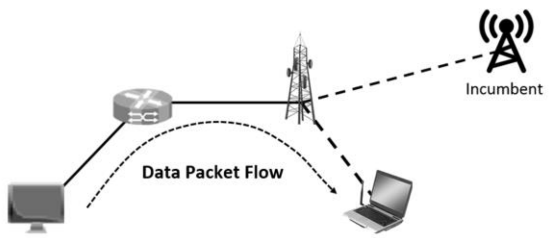

Fig. 1. The system configuration

The scenario analyzed is shown in Fig. 1. The scenario consists of data transfer from a wired node to a CPE through a router and a BS. An incumbent is also present whose *"keep out distance"* can be changed according to requirement. The bandwidth of each channel is equal to 6 MHz with 60 subchannels in accordance to the 802.22 standard. The probability of detection is presumed to be one and the probability of false alarm zero. The modulation used is 16QAM with the cyclic prefix factor (*CP*) ¼ and coding rate (*FECr)* ½. UDP Protocol is used in Transport Layer.

B. Design Options

1) Incumbent: We have one incumbent in our scenario and clearly interference with an incumbent reduces the throughput of the SU. But we need to determine how the incumbent is modelled. One strategy is to set the distance between the incumbent and the CPE more than the *"keep out distance"*, to avoid any interference. We call this strategy I_0. Another strategy is to keep the busy time and idle time of the incumbent constant.We call this strategy I_1. The last strategy is the one in which the busy times and idle times of the incumbent follow negative exponential distribution with their respective known means. We call this strategy I_2.Note that for strategies I_1 and I_2, we assume the CPE is within the *"keep out distance"* of the incumbent. Thus we have three design options.

V. PROPOSED ANALYTICAL MODEL

A. Case I_0

Let us first consider the case when we don't have any interference. To ensure this, we keep the CPE outside the *"keep out distance".* We set the Packet size *P* as 58 bytes. The packets are generated with an inter-arrival time $\Delta_{p,a}$ of 200us. The headers involved are 8 bytes for UDP, 20 for IP and 4 for MAC. So the total overhead per packet H is 32 bytes. This leads to a total packet size of $P + H = 90$ bytes. The reason for choosing this packet size is explained below.

The number of data sub carriers in a channel N_D is fixed by the 802.22 standard as 1440. The gross bits conveyed per symbol per subcarrier (*N)* is 4. Thus the total number of bytes per OFDMA symbol is $N_D * N * FECr / 8 = 360$ bytes, which is an integral multiple of ($P + H$). So an integral number of data packets are transmitted per OFDMA symbol, which simplifies our analysis.

Let t_f be the frame duration. Then, the number of symbols per frame is given by:

$$S = \{t_f - (TTG + RTG)\} / T_{SYM} \quad (1)$$

where T_{SYM} is the symbol duration given by the 802.22 standard.

We set the *Allocation Start Time* to be such that we have an equal number of downstream and upstream symbols in each frame. So total numbers of DS symbols in a frame are:

$$S_{DS} = S - \text{Allocation Start Time} \quad (2)$$

Note that the number of downstream symbols for data packets is lesser than S_{DS} by 4 for the first frame and by 2 for other frames in any superframe. This is because of the downstream symbols reserved for control packets, SCH, superframe preamble and frame preamble. Let f be the number of frames in a superframe. The total number of symbols in a superframe is therefore

$$\bar{S}_{DS} = 1 \times (S_{DS} - 4) + (f - 1) \times (S_{DS} - 2) = S_{DS} \times f - 34 \quad (3)$$

The maximum data rate which can be obtained using a single channel in downstream direction is given by:

$$C_0 = (N_D \times N \times FECr \times \bar{S}_{DS}) / (f \times t_f) \qquad (4)$$

Let X be the total number of channels. Then the total bandwidth available for downstream transfer is:

$$B_0 = C_0 \times X. \qquad (5)$$

The rate at which data is coming from the application is

$$\nu_a = 8 \times P / \Delta_{p,a} \qquad (6)$$

The net data rate (including headers) is given by:

$$\nu_n = \nu_a \times (P + H) / P. \qquad (7)$$

The gross throughput which can be obtained from the network is $\Theta_{g0} = \text{Min}(\nu_n, B_0)$. Assuming $B_0 < \nu_n$,

$$\Theta_{g0} = B_0. \qquad (8)$$

The useful throughput is given by:

$$\Theta_{d0} = \Theta_{g0} \times P/(P+H) \qquad (9)$$

Substituting all the variables, we obtain the final formula as:

$$\Theta_{d0} = \frac{(N_D \times N \times FECr \times \bar{S}_{DS}) \times X \times P}{(f \times t_f) \times (P+H)} \qquad (10)$$

B. Case I_1

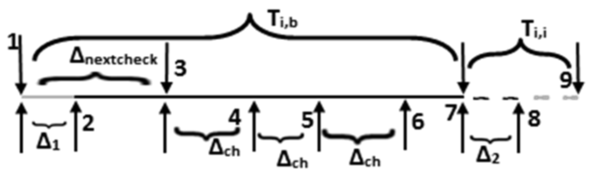

Fig. 2. (Not to scale) Events during a cognitive radio transmission cycle for incumbent with constant ide/busy periods. The dark horizontal line indicates that the SU transmission is OFF, while light line indicates that the SU transmission is ON. The solid line indicates incumbent is busy, while the dotted line indicates incumbent is idle.Please refer to TABLE I for details

TABLE I. EXPLANATION OF FIG. 2

Event Id	Time (s)	Description
1	x	Incumbent becomes busy
2	$x + \Delta_1$	Transmission stops
3	$x + \Delta_{nextcheck}$	SCH transmission fails CPE checks for a free channel but cannot find any
4	$x + \Delta_{nextcheck} + 1\Delta_{ch}$	CPE checks for a free channel but cannot find any
5	$x + \Delta_{nextcheck} + 2\,\Delta_{ch}$	
6	$x + \Delta_{nextcheck} + 3\,\Delta_{ch}$	
7	$x + T_{i,b}$	Incumbent becomes idle
8	$x + T_{i,b} + \Delta_2$	CPE checks for a channel and finds one; transmission restarts
9	$x + T_{i,b} + T_{i,i}$	Incumbent becomes busy

The busy and idle periods of the incumbent are constant as described in section IV. B. 1) case I_1.

We keep the packet size and headers same as in section *A*. So (1) - (3) are still valid for this case. Now we need to consider the percentage of time for which the incumbent blocks the transmission. We assume that the incumbent is idle for $T_{i,i}$ = 4s and then busy for an equal amount of time (i.e. $T_{i,b}$ = 4s too) . This cycle then keeps on repeating. The reason for keeping the busy and idle times as 4s is that it will cover an integral number of superframes (each of 0.16s), which will make our analysis simpler.

Consider any time x, when an incumbent has just begun operation.

Clearly, $x \in \{ T_{i,i} + n * (T_{i,i} + T_{i,b}), n \in \mathbb{N} \}$

The sequence of events which occurs between $[x, x + (T_{i,i} + T_{i,b})]$ repeats itself in $[x + (T_{i,i} + T_{i,b}), x + 2 \times (T_{i,i} + T_{i,b})]$ and so on. So it is sufficient to consider this period for our analysis.

After the incumbent becomes busy at x, it takes one frame for the CPE to detect it and another frame to convey this information to the BS, which switches OFF the transmission. So, there is a time-lag of $\Delta_1 = 2t_f$ in detecting a change in the incumbent state.

A similar time-lag Δ_2 occurs in detecting that state of the incumbent has become idle. The cause of this lag is that the SU senses the channels only after an interval of Δ_{ch} = 1s. The SU does not sense the channel continuously. This sensing begins when an SCH fails to transmit. Look at Fig. 2 for more details.

Note that these time-lags, which are the delays in detection of change of state of incumbent by the SU, form a very crucial part of our analysis.

Δ_2 will depend upon $\Delta_{nextcheck}$, which is the delay in checking of a free channel by the SU for the first time with respect to the time when incumbent becomes busy. This check, in our model, is done at the next scheduled transmission of SCH just after the incumbent becomes busy.

$$\text{Let } \Delta_3 = (x + T_{i,i} + T_{i,b}) - (x + T_{i,b} + \Delta_{nextcheck})$$
$$= T_{i,i} - \Delta_{nextcheck}$$

Then $\Delta_2 = \lceil \Delta_3 / \Delta_{ch} \rceil \times \Delta_{ch} - \Delta_3$

As evident in Fig. 2, the transmission by the BS is blocked during $[x + \Delta_1, x + T_{i,b} + \Delta_2]$, i.e. for $(T_{i,b} + \Delta_2 - \Delta_1)$ and therefore, transmission occurs only for $(T_{i,b} + T_{i,i}) - (T_{i,b} + \Delta_2 - \Delta_1) = (T_{i,i} - \Delta_2 + \Delta_1)$ out of $(T_{i,b} + T_{i,i})$. So the fraction of time for which the transmission occurs in case *B* is:

$$\alpha_1 = (T_{i,i} - \Delta_2 + \Delta_1)/(T_{i,b} + T_{i,i}) \qquad (11)$$

Neglecting the difference in amount of data transferred in the first frame as compared to any other frame, and hence assuming a uniform data rate over all frames, the maximum data rate which can be obtained using a single channel in downstream direction is:

$$C_1 = \alpha_1 \times C_0 \qquad (12)$$

Equations (5)-(9) are valid for this case too with a change in the subscripts of variables from 0 to 1 to signify change from case *A* to *B*. Following the same analysis, we get the new useful throughput as:

$$\Theta_{d1} = \alpha_1 \times \Theta_{d0} \quad (13)$$

The throughput decreases by a factor of α_1.

C. *Case I_2*

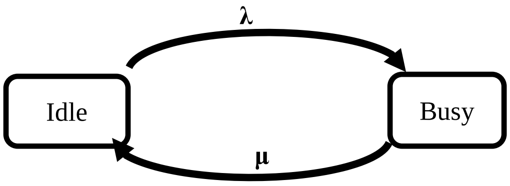

Fig 3. The continuous time markov chain representation of the incumbent operation with idle rate λ and busy rate μ.

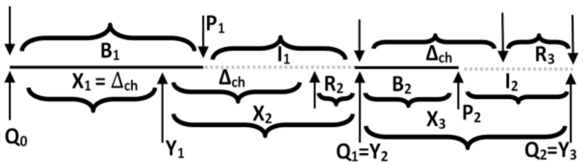

Fig 4. Events during a cognitive radio transmission cycle: for incumbent when ide and busy time of the incumbent follow exponential distribution. Different cycles are labelled as X_i. B_i and I_i are the durations of the i[th] busy and idle periods respectively. R_i represents the reward in the i[th] cycle. X_1 belongs to case 1 (so R_1 =0), while X_2 and X_3 belong to case 2 (they have nonzero rewards).

With reference to Section IV B. 1) case I_2, we will now define the model used in this case. We model the incumbent as continuous-time markov chain with two possible states- busy and idle, as shown in Fig 3. The rate of transition from idle state of the incumbent to busy state is λ; while the rate of the transition in opposite direction is μ. We assume that the incumbent is initially busy for the purpose of our theoretical analysis; note that however, in simulation, the idle period of the incumbent occurs first.

The SU stops transmission as soon as it finds the channel busy. Then, it scans the channel after every Δ_{ch} = 1s. One approximation of the theoretical model is that the scans begin immediately after the incumbent changes state to busy from idle; while in simulation (and in the real world), the scans begin at the starting of the next superframe just after the incumbent becomes busy. The transmission is restarted as soon as the channel is found idle in one of the scans and continues till the next busy period.

Although we have instantaneous sensing, we will refer to the gap between two consecutive sensing as "*sensing period'* for ease of writing.

Let B_i and I_i be the durations of the i[th] busy and idle periods respectively. Now we define the following quantities:

$$X_1 = \begin{cases} \Delta_{ch} \quad , & if\ \Delta_{ch} < B_1 \\ B_1 + I_1, & otherwise \end{cases}$$

We assume that $X_1 \leq B_1 + I_1$.

Let P_i denote the time when i[th] busy period ends and Q_i denote the time when i[th] idle period ends. So $P_1 = B_1$ and $Q_0 = 0$. X_i.. is the duration of i[th] cycle. Y_i denotes the time at the end of i[th] cycle. So, $Y_0 = 0$. Now we define X_i as:

$$X_i = \begin{cases} \Delta_{ch} \quad , & if\ \Delta_{ch} + Y_{i-1} < P_{S_i} \text{(We say that the } i^{th} \text{ cycle is of type } a) \\ B_{S_i} + I_{S_i}, & otherwise \text{ (We say that the } i^{th} \text{ cycle is of type } b) \end{cases}$$

Here $S_i = sup\{n :\ Q_{n-1} \leq Y_{i-1}\}$. This means that S_i is the index of the last busy period which starts before the starting of the i[th] cycle.

We assume that $\Delta_{ch} + Y_{i-1} \leq Q_{S_i}$. In other words, sensing duration Δ_{ch} is always smaller than the combined length of one busy and one idle period. This assumption is justified because the mean idle time and the mean busy time of the incumbent are quite large as compared to Δ_{ch}.

We call the time for which SU transmission occurs in each cycle is called the reward of that cycle. Reward in i[th] cycle is denoted by R_i. First few cycles and rewards are shown in Fig. 4.

Without loss of generality, let us analyze the first cycle.. The probability that the first cycle is of type *a* is:

$$\Pr(B_1 \geq \Delta_{ch}) = 1 - F_{B_1}(\Delta_{ch}) = e^{-\mu\Delta_{ch}} \quad (14)$$

Here $F_{B_1}(x)$ is the cumulative frequency distribution for B_1, which is $e^{-\mu x}$. Note that the reward is zero in this case as the transmission was completely blocked during the cycle. (In Fig 4, the first cycle is depicted as type *a* cycle).

The probability that the first cycle is of type *b* is:

$$\Pr(B_1 < \Delta_{ch} \leq B_1 + I_1) \approx 1 - Pr(B_1 \geq \Delta_{ch}) = 1 - e^{-\mu\Delta_{ch}} \quad (15)$$

A type *b* cycle ends when the idle period ends, i.e. $X_1 = B_1 + I_1$. The transmission will begin when the channel is detected idle at the second sensing and, will continue till the end of the idle period, i.e. $R_1 = B_1 + I_1 - \Delta_{ch}$.

These results are tabulated in the Table II

TABLE II. TWO CASES FOR SENSING TIME

Cycle type	Condition	Probability	X_1	R_1
a	$\Delta_{ch} \leq B_1$	Exp(-μΔ_{ch})	Δ_{ch}	0
b	$B_1 < \Delta_{ch} \leq B_1 + I_1$	≈ 1- Exp(-μΔ_{ch})	$B_1 + I_1$	$B_1 + I_1 - \Delta_{ch}$

It can be shown that all X_i*'s* are independent identically distributed variables, and so are the R_i*'s.* Let W_t be the total reward received till time *t.* Then by the *"elementary renewal theorem for renewal reward processes"*, we get:

$$\lim_{t\to\infty} \frac{W_t}{t} = \frac{\mathrm{E}[R_1]}{\mathrm{E}[X_1]}$$

Let α_2 be the fraction of time for which SU transmission occurs.

$$\text{Then } \alpha_2 = \lim_{t\to\infty} \frac{W_t}{t}$$

$$\Rightarrow \alpha_2 = \frac{E[R_1]}{E[X_1]} \quad (16)$$

$$E[X_1] = \Pr(\Delta_{ch} \le B_1) \times \Delta_{ch} + \Pr(B_1 < \Delta_{ch} \le B_1 + I_1) \times E[B_1 + I_1 \mid B_1 < \Delta_{ch} \le B_1 + I_1] \quad (17)$$

$$E[R_1] = \Pr(\Delta_{ch} \le B_1) \times 0 + \Pr(B_1 < \Delta_{ch} \le B_1 + I_1) \times E[B_1 + I_1 - \Delta_{ch} \mid B_1 < \Delta_{ch} \le B_1 + I_1] \quad (18)$$

(17) and (18) follow from the definition of expectation.

Let us calculate the conditional expectation involved in (18).

$$E[B_1 + I_1 - \Delta_{ch} \mid B_1 < \Delta_{ch} \le B_1 + I_1]$$
$$= \int_0^\infty \{1 - F_{((B_1 + I_1 - \Delta_{ch}) \mid (B_1 < \Delta_{ch} \le B_1 + I_1))}(r)\} dr \quad (19)$$

$$\text{Let } g(r) = 1 - F_{((B_1 + I_1 - \Delta_{ch}) \mid (B_1 < \Delta_{ch} \le B_1 + I_1))}(r) \quad (20)$$

$$\Rightarrow g(r) = \Pr(B_1 + I_1 - \Delta_{ch} > r \mid B_1 < \Delta_{ch} \le B_1 + I_1)$$

$$\Rightarrow g(r) = \frac{\Pr(B_1 + I_1 > \Delta_{ch} + r \cap B_1 < \Delta_{ch})}{\Pr(B_1 < \Delta_{ch} \le B_1 + I_1)} \quad (21)$$

$$\Pr(B_1 + I_1 > \Delta_{ch} + r \cap B_1 < \Delta_{ch})$$
$$= \int_0^{\Delta_{ch}} \mu e^{-\mu x} \left(\int_{\Delta_{ch} + r - x}^{\infty} \lambda e^{-\lambda y} dy \right) dx$$
$$= \int_0^{\Delta_{ch}} \mu e^{-\mu x} \left\{ \left[e^{-\lambda y} dy \right]_{\infty}^{\Delta_{ch} + r - x} \right\} dx$$
$$= \int_0^{\Delta_{ch}} \mu e^{-\mu x} e^{-\lambda(\Delta_{ch} + r - x)} dx$$
$$= \mu e^{-\lambda(\Delta_{ch} + r)} \int_0^{\Delta_{ch}} e^{(\lambda - \mu)x} dx$$
$$= \mu e^{-\lambda(\Delta_{ch} + r)} \left[e^{(\lambda - \mu)x} \right]_0^{\Delta_{ch}}$$
$$= \mu e^{-\lambda(\Delta_{ch} + r)} \frac{\left[e^{(\lambda - \mu)\Delta_{ch}} - 1 \right]}{(\lambda - \mu)} \quad (22)$$

$\Pr(B_1 < \Delta_{ch} \le B_1 + I_1)$ is a special case of (22) with $r = 0$.So,

$$\Pr(B_1 < \Delta_{ch} \le B_1 + I_1) = \mu e^{-\lambda(\Delta_{ch})} \frac{\left[e^{(\lambda - \mu)\Delta_{ch}} - 1 \right]}{(\lambda - \mu)}. \quad (23)$$

Substituting (22) and (23) in (21), we get:

$$g(r) = \frac{\mu e^{-\lambda(\Delta_{ch} + r)} \frac{\left[e^{(\lambda - \mu)\Delta_{ch}} - 1 \right]}{(\lambda - \mu)}}{\mu e^{-\lambda(\Delta_{ch})} \frac{\left[e^{(\lambda - \mu)\Delta_{ch}} - 1 \right]}{(\lambda - \mu)}.}$$

$$\Rightarrow g(r) = e^{-\lambda r} \quad (24)$$

From (19), (20) and (24), we have:

$$\Rightarrow E[B_1 + I_1 - \Delta_{ch} \mid B_1 < \Delta_{ch} \le B_1 + I_1]$$
$$= \int_0^\infty e^{-\lambda r} dr = 1/\lambda \quad (25)$$

$$\Rightarrow E[B_1 + I_1 \mid B_1 < \Delta_{ch} \le B_1 + I_1]$$
$$= \Delta_{ch} + E[B_1 + I_1 - \Delta_{ch} \mid B_1 < \Delta_{ch} \le B_1 + I_1]$$

$$\Rightarrow E[B_1 + I_1 \mid B_1 < \Delta_{ch} \le B_1 + I_1] = \Delta_{ch} + 1/\lambda \quad (26)$$

(26) follows from (25) due to linearity of expectation.

Substituting (14), (15) and (26) in (17), we get:

$$E[X_1] = e^{-\mu\Delta_{ch}} \times \Delta_{ch} + (1 - e^{-\mu\Delta_{ch}}) \times (\Delta_{ch} + 1/\lambda)$$

$$\Rightarrow E[X_1] = (\Delta_{ch} + 1/\lambda) - e^{-\mu\Delta_{ch}} \times 1/\lambda \quad (27)$$

Substituting (14), (15) and (25) in (18), we get:

$$E[R_1] = e^{-\mu\Delta_{ch}} \times 0 + (1 - e^{-\mu\Delta_{ch}}) \times 1/\lambda$$

$$E[R_1] = (1 - e^{-\mu\Delta_{ch}}) \times 1/\lambda \quad (28)$$

Substituting (27) and (28) in (26), we get:

$$\alpha_2 = \frac{(1 - e^{-\mu\Delta_{ch}}) \times 1/\lambda}{(\Delta_{ch} + 1/\lambda) - e^{-\mu\Delta_{ch}} \times 1/\lambda}$$

$$\Rightarrow \alpha_2 = \frac{(1 - e^{-\mu\Delta_{ch}})}{(\lambda\Delta_{ch} + 1) - e^{-\mu\Delta_{ch}}} \quad (29)$$

Neglecting the difference in amount of data transferred in the first frame and any other frame, and hence assuming a uniform data rate over all frames, the maximum data rate which can be obtained using a single channel in downstream direction is:

$$C_2 = \alpha_2 \times C_0 \quad (15)$$

Equations (5)-(9) are valid for this case too with a change in the subscripts of variables from 0 to 2 to signify change from case *A* to *C*. Following the same analysis, we get the new useful throughput as:

$$\Theta_{d2} = \alpha_2 \times \Theta_{d0} \quad (16)$$

The throughput thus decreases by a factor of α_2.

VI. NUMERICAL RESULTS AND DISCUSSION

In this section, the values used for throughput calculation are tabulated case-wise. They are accompanied by the graphs of throughput obtained through simulation ran in NetSim v8.3, shown in Fig (5)-(7).

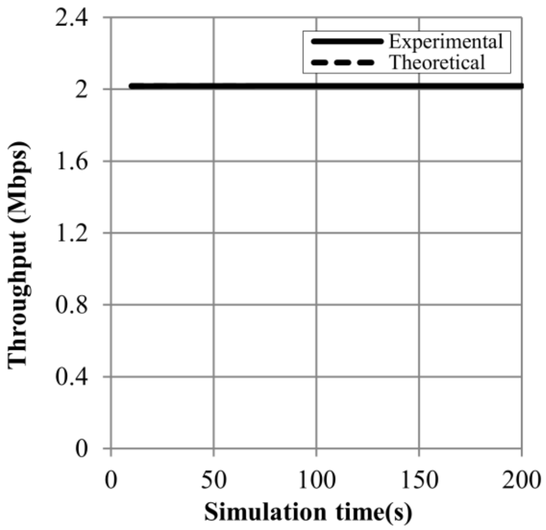

Fig 5. Throughput forc I_0: no incumbent

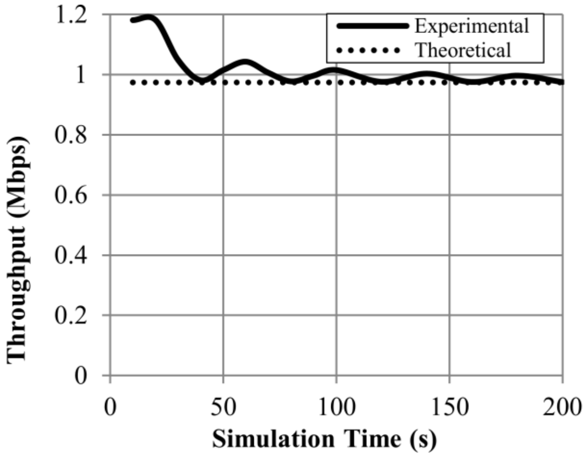

Fig 6. Throughput for case I_1: incumbent with constant busy and idle periods.

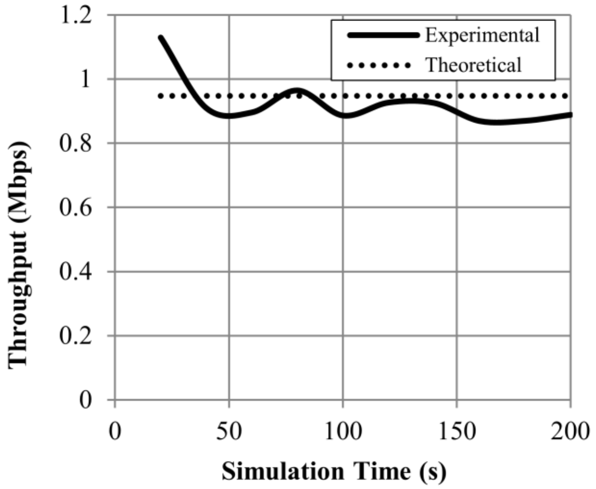

Fig 7. Throughput for case I_2: incumbent with exponentially distributed busy and idle periods.

TABLE III. NETSIM SIMULATION PARAMETERS

Parameter	**Value**		
	Case *A*(*i*=0)	Case *B*(*i*=1)	Case *C*(*i*=2)
P	58 bytes		
H	32 bytes		
$\Delta_{p,a}$	200 us		300 us
N_D	1440		
N	4		
$FECr$	1/2		
$\bar{S}_{DS}$	174		
X	1		
f	16		
t_f	10 ms		
$T_{i,i}$	∞	4 s	4 s
$T_{i,b}$	0	4 s	4 s
α_i	1.0000	0.4825	0.4694
Θ_{di}	2.0184 Mbps	0.9739 Mbps	0.9475 Mbps

VII. CONCLUSION

In this paper, three models of incumbent operation have been analyzed. One of the important issues that the paper focuses on, is the lags that are involved in detection (by the SU) that the incumbent has changed its state between idle and busy. One key observation is that the throughput obtained from the theory compares well against the steady state simulation result.

Although the paper focuses on the case with one incumbent and one channel only, we can extend our analysis to include more channels and/or incumbents. Other modifications include analysis of channel bonding and/or notching

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